Reassessment of 3D synchro-resonant detector for high-frequency gravitational waves

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Abstract

Previously reported results for a gravity wave detector regarding photon generation arising from the interaction between an incident high-frequency gravitational wave, a locally-generated Gaussian electromagnetic beam, and a local steady magnetic field, are reassessed using symmetry and a rigorous interpretation of Poynting's theorem. The conclusions are that transverse photon generation in this geometry is impossible, and that wave impedance cannot be used as the basis for distinguishing weak photon generation from other spuriae.

Keywords: photon generation, gravitational wave detection, wave impedance, Li-Baker detector, HFGW detection

1. Introduction

The conceptual design of an instrument for detection of high-frequency gravitational-waves

(HFGWs) has recently risen to prominence following confirmed detection of low-frequency GWs. One possibility that has been investigated both theoretically and experimentally is to use a microwave waveguide system exploiting the rotation of the polarization vector of an electromagnetic (EM) wave in the curved spacetime caused by the passage of a gravitational wave [1]. This has been found to have insufficient sensitivity for detection of the most reliable HFGW sources expected to exist from dependable predictions, the relic HFGW signatures arising from past cataclysmic cosmological events such as the Big Bang [2, 3, 4].

Another very general proposed method of HFGW detection is to measure the oscillatory EM field that results from a HFGW propagating through a region of space in which there is a superimposed static (and nominally uniform) magnetic field. The oscillatory EM field is produced as a result of the inverse Gertsenshte±n effect [5]. In principle, the effect may be enhanced by using a high-*Q* resonant microwave cavity [6], though in these arrangements also the predicted sensitivity remains significantly poorer than is necessary to detect cosmological HFGW sources.

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reportedly causes photon emission (Perturbative Photon Flux, PPF) along the *x*-axis [7]. The present work re-examines PPF generation from first principles.

2. Gaussian beam

The field distribution in an EM GB [8] is

$$\psi = \frac{\psi_0}{\sqrt{1+(z/f)^2}} \exp\left(-\frac{r^2}{W^2}\right) \exp\left[i\left(k_{EM}z - \omega_{EM}t\right) - \arctan\left(\frac{z}{f}\right) + \frac{k_{EM}r^2}{2R} + \delta\right] \qquad (1)$$

where ψ signifies a component of either the electric or magnetic field, ψ_0 is its maximum value at $r = z = 0$, ω_{EM} is the angular frequency, $r^2 = x^2 + y^2$, the EM wavenumber is $k_{EM} = 2\pi/\lambda_{EM}$, $W = W_0\sqrt{[1 + (z/f)^2]}$ is the cross-section radius of the GB at a given value of z, W_0 is the minimum cross-section radius of the GB, the parameter $f = \pi W_0^2/\lambda_{EM}$, $R = z + f^2/z$ is the curvature radius of the GB wave-front at z, and δ is an arbitrary phase shift. Note that the width W of the GB diverges as $|z|$ increases (as exp3ou0e

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Figure 1. Transverse BPF (FF) and transverse PPF (FF) for $z > 0$, and GB envelope and local propagation direction (FF x), redrawn from Fig. 2 of Ref. [11], Fig. 6 of Ref. [12], and Fig. 5 of Ref. [14].

3.1 Poynting's theorem

Use of the Poynting vector (×) to calculate photon flux requires considering a rigorous statement of Poynting's theorem [15]:

$$- \quad (\quad \times \quad) \cdot \mathrm{d} \quad = - \int \quad (\qquad \text{energy density}) \mathrm{d} \quad + \int \quad \cdot \quad \mathrm{d} \qquad (3)$$

(strictly, for quasi-static conditions or over small volumes to avoid complications due to the finite speed of radiative energy transfer). No further significant rigorous progress is possible in general, for example to find the precise meaning of (×). The frequent interpretation that "the Poynting vector" (×) is equal to power flow per unit area is consistent with Eqn. (3).*

* A simple counter-example is as follows. Suppose that in free space a constant, uniform, electric field 100kV/m is established parallel to the *x*-axis, and a constant, uniform, magnetic field of H = 800kA/m ($B \approx 1$T) is established parallel to the *y*-axis. Both these fields are read,gne3icnsii, nth

However, this is analogous to conjecturing that if $\int_0^1 (\quad)\mathrm{d}\quad = 1$ then it must follow that $y(x) = 1$; this is one possible solution, but there are infinitely many more [16]. It follows that universally assigning $(\quad \times \quad)/=\omega_{\mathrm{EM}}$ unequivocally and indiscriminately as BPF is far from certain or proven in general, and is potentially incorrect. However, arguments involving symmetry allow some significant progress to be made in the present case.

3.2 Symmetry

the z-axis for all negative z, all as expected by symmetry. (The integration result does not prove that the photon flux in the x-direction and y-direction is zero for $z \neq 0$. A photon flux from infinite r directed towards the z-axis for $z < 0$ is not impossible *a priori* as the "power source at infinity" is the GB itself extending to infinity.) By symmetry, the horizontal photon flux should flip polarity for any of x, y, z changing polarity. However, the conclusion is that the horizontal BPF, $n_x^{(0)}$ and $n_y^{(0)}$, is not given correctly by evaluating Poynting's vector as it does not exhibit the correct (x, y, z) symmetry. The correct expression is calculated in the Appendix.

4. Perturbative Photon Flux

Figure 2 shows the essential previously proposed features of a 3DSR HFGW detector. A constant magnetic flux density $\underline{B}_y$ (straight underline denotes constant values) is applied in the y-direction, normal to the propagation direction of the HFGW to be detected. Interaction between any incident HFGW, the magnetic field, and the GB takes place over $-\Delta l/2 < z < +\Delta l/2$ (i.e., interaction length Δl). Other features such as superconducting cavity walls, and a fractal membrane for focusing the output signal, vary in different postulated implementations of the 3DSR and are merely refinements intended to maximize the sensitivity obtained from the basic setup. These may be safely ignored here as they do not affect the fundamental geometry shown in Fig. 2 or the fundamental field interactions that depend solely upon the spatial directions of the fields concerned (in addition to other design parameters such as frequency of operation).

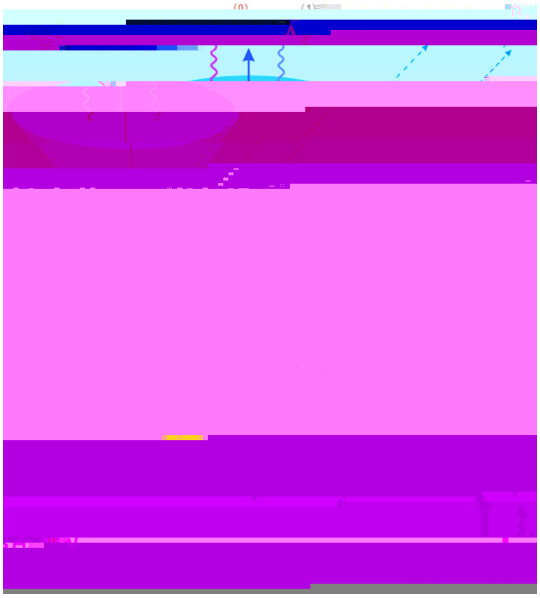

Figure 2. Diagram of proposed 3DSR HFGW detector. The system consists of a GB photon flux directed along the +*z*-axis, a constant magnetic field applied parallel to the *y*-axis, and a fractal membrane [17] or equivalent microwave lens or mirror (not shown) intended to collect and focus the signal photon flux or PPF produced from the components of the incident HFGW phase-coherent with the GB ("synchro-resonance"). The propagation directions of $n_x^{(0)}$ for $z > 0$ and for $z < 0$ are opposite.

The incident HFGW may be treated as a spectrum composed of plane waves of infinite extent and of varying directions and frequencies as no significant local sources of HFGW are known and the cataclysmic cosmic sources of HFGW are, by definition, located at infinity. The perturbed EM fields (indicated by superscript (1)) produced by direct interaction of an incident HFGW with the constant magnetic flux density $\underline{B}_y$ are given by [9, 18]

$$\begin{aligned} \underline{E}_y^{(1)} &= -\tfrac{1}{2} h \underline{B}_y c k_{\mathrm{GW}} \Delta l \exp\left(i\left[k_{\mathrm{GW}} z - \omega_{\mathrm{GW}} t\right]\right), \\ \underline{B}_x^{(1)} &= +\tfrac{1}{2} h \underline{B}_y k_{\mathrm{GW}} \Delta l \exp\left(i\left[k_{\mathrm{GW}} z - \omega_{\mathrm{GW}} t\right]\right), \end{aligned} \tag{4}$$

where h is the HFGW amplitude. Here, $\underline{E}_y^{(1)}$ Δl and $\underline{B}_x^{(1)}$ Δl because GWs and generated EM waves have the same propagation velocity in a vacuum, so that the two waves are phase-coherent if they have the same frequency and direction, and the effect accumulates proportional to the interaction length [9, 19]. The perturbative EM fields propagating in the negative z-direction (i.e., opposite to the HFGW) are neglected, because they are much weaker or absent [9, 18, 19]. Moreover, as $\underline{E}_y^{(1)}$ $ck_{\mathrm{GW}} = \omega_{\mathrm{GW}}$ and $\underline{B}_x^{(1)}$ $k_{\mathrm{GW}} = \omega_{\mathrm{GW}}/c$, the interaction cross-section is larger for higher frequency. The resultant perturbative EM power flux in the z-direction ($= -\underline{E}_y^{(1)}\underline{B}_x^{(1)}/\ _0$) from Eqn. (4) is proportional to the square of the HFGW amplitude h; this is the classical inverse Gertsenshtein effect [5]. However, the relic HFGW amplitude predicted from typical cosmological models [2, 3, 4] is at best $h \approx \sim 10^{-30}$ to $\sim 10^{-29}$ in the GHz band. (This compares to experimental upper limits of $h \approx 6 \times 10^{-26}$ at (2.7 to 14) $\times 10^5$ GHz and $h \approx 5 \times 10^{-28}$ at (5 to 12) × 10s5 (r6Tcal)5.9 bTdbservtudund6 (labo) T80(rato) T80(ry condi Td) T80(s1 (pares t>>BDC 0.0

components, or the HFGW spectral components satisfying $\omega_{\mathrm{EM}} = \omega_{\mathrm{GW}}$ within the precision implied by the overlap integral of the HFGW and EM wave over interaction length Δl), is then [12, 22]

$$n_x^{(1)} \quad \frac{c}{\mathrm{EM}} \left\langle T^{01(1)} \right\rangle_{\omega_{\mathrm{EM}} \ \omega_{\mathrm{GW}}}$$

$$\frac{1}{\mathrm{EM}} \left\langle \frac{E_y^{(1)} B_z^{(0)}}{0} \right\rangle_{\omega_{\mathrm{EM}} \ \omega_{\mathrm{GW}}}$$

$$= \hat{u}\, l \frac{\underline{B}_y h H_{z,0}}{4 \sqrt{1 + (z/f)^2}} \exp \quad \frac{r^2}{W^2} \ \cos \ \frac{k_{\mathrm{GW}} r^2}{2R} \quad \arctan \ \frac{z}{f} \ +$$

4.1 Waves in Poynting's theorem

For *infinite-extent plane sinusoidal* EM waves, evaluating (×) by replacing the instantaneous values of **E** and **H** by their respective amplitudes gives *twice* the *average* power flow per unit area of the phase-fronts in the correct direction. However, that is only the case when both **E** and **H** vary sinusoidally, in phase coherence, and perpendicular to the propagation direction, such as $\mathbf{E} = \mathbf{E}_o\exp(i[k_{EM}z - \&_{EM}t] + /)$ and $\mathbf{H} = \mathbf{H}_o\exp(i[k_{EM}z - \&_{EM}t] + /)$. But in Eqn. (5), $\underline{B}_z^{(0)}$ is one of the original field components of the GB given by Eqn. (1), while $\underline{E}_y^{(1)}$ is a perturbation given by Eqn. (4). Both of these contain the fundamental variation $\exp(i[k_{GW}z - \&_{GW}t] + /)$ which has no *x*- or *y*-axis periodicity. In fact Eqn. (4) for $\underline{E}_y^{(1)}$ contains no dependence upon *x* or *y*, while Eqn. (1) for $\underline{B}_z^{(0)}$ contains non-sinusoidal dependence upon *x* and *y* proportional to

$\exp(-r^2/W^2 + ik_{EM}r^2/2R) = \exp(-[x^2 + y^2]/W^2 + ik_{EM}[x^2 + y^2]/2R)$ which is also shown in Eqn. (5) and is aperiodic in *x* and *y* as $R \rightarrow \infty$. The question is whether this corresponds to photon generation and propagation within a horizontal plane. The variable oscillation proportional to $\exp(ik_{EM}r^2/2R) = \Psi$ implies a wavevector horizontal component given by definition as $-i(\partial\Psi/\partial r)/\Psi = k_{EM}(r/R)$, which is proportional to the radius from the *z*-axis. This in turn implies either a frequency change in propagating away from the *z*-axis, or a curved propagation path (which could give a varying in-plane wavevector component even if the wavenumber magnitude is constant). Both of these are impossible for MC /adius f

PPF in the horizontal plane with no contradiction if $\underline{E}_y^{(1)}\underline{B}_z^{(0)}$ is interpreted rigorously in Poynting's theorem and integrated again over the infinitely high cylindrical surface $x^2 + y^2$ = constant. Although $\underline{E}_y^{(1)}\underline{B}_z^{(0)}$ is non-zero, the result of that surface integration is zero by symmetry, consistent with zero photon flux in any horizontal direction arising from the product $\underline{E}_y^{(1)}\underline{B}_z^{(0)}$. The conclusion is that photon generation and propagation described as PPF is impossible using the previously described geometry.

5. Wave impedance

The EM wave impedance is defined as the electric field amplitude ($\underline{E}$) divided by the magnetic field strength amplitude ($\underline{H}$) of an EM wave, and in free space it is well-known that the wave impedance for a plane EM wave [23] is $_0 = 377$. Equations (1) and (4) give the claimed wave impedance for the transverse PPF in the *x*-direction in the 3DSR system as [24]

$$Z = \frac{-\underline{E}_y^{(1)}}{\underline{H}_z} \approx \Delta l \frac{h\underline{B}_y^{(0)}\omega}{2\underline{H}_{z,0}} \sqrt{1+(z/f)^2} \exp\left(\frac{r^2}{W^2}\right) \quad (6)$$

Estimated values of this expression for typical 3DSR system design parameters and likely incident HFGW properties range from 2×10^{-4} down to 10^{-19} for the PPF [24], a tiny fraction of $_0$. The enormous difference between these PPF values and $_0$ has been used to suggest that impedance matching techniques might be used to distinguish between PPF and BPF.

5.1 Impedance value

For a high-amplitude infinite plane wave propagating parallel to the +*z*-axis, with $\underline{E}_y = \underline{H}_x = 0 = \underline{E}_z = \underline{H}_z$, and non-zero electric field $\underline{E}_x$ and magnetic field strength $\underline{H}_y$, clearly $\underline{E}_x$ and $\underline{H}_y$ are related by $Z = \underline{E}_x/\underline{H}_y = _0 = 377\Ÿ$.

Now superimpose a small-amplitude infinite plane wave at the same frequency propagating

effect and a GB, it is impossible on symmetry grounds to generate a PPF perpendicular to the directed GB using the geometry previously proposed, irrespective of the use of a superconducting cavity or fractal membrane for sensitivity enhancements. The wave impedance of the PPF is the same as the free-space impedance, and cannot be used as the basis of distinguishing PPF from other waves.

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Appendix: Horizontal BPF from GB

For comparison with Eqn. (2) and figure 1, the correct expression for the horizontal photon flux from a conventional TEM GB is calculated by considering photon flow in and out of a thin disc between horizontal planes $z = Z$ and $z = Z + dz$ and within a cylindrical surface of radius $! = \sqrt{(x^2 + y^2)}$. The photon flow rate per unit area impinging on every point of the plane $z = Z$ is given by applying Poynting's theorem (Eqn. (3)) to the fields given by Eqn. (1) (corresponding to a source at $z = 0$), rigorously within the volume defined by the plane $z = Z > 0$ and the surface at infinity for all $z < Z$. This gives

$$^{(0)} = \frac{^{(0)}_{,0}}{1+(\ /\ )^2}\exp(-2\ ^2/\quad ^2). \tag{A1}$$

Therefore, the total photon flow rate into the disc through its lower plane face is

$$= \int_0 \frac{^{(0)}_{,0}}{1+(\ /\ )^2}\exp(-2\ ^2/\quad ^2)2\quad \mathrm{d} \tag{A2}$$

$$= \frac{^{(0)}_{,0}\quad ^2}{2(1+\ ^2/\ ^2)}[1-\exp(-2\ ^2/\quad ^2)] \tag{A3}$$

$$= \tfrac{1}{2}\ ^{(0)}_{,0}\quad\ ^2_0[1-\exp(-2\ ^2/\quad ^2)]. \tag{A4}$$

This is a function of *z* through the *z*-dependence of *W*, but it becomes independent of *z* as !→ ∞ (as expected). The total photon flow rate out of the disc through its upper plane face is the same as given by Eqn. (A4) plus the difference given by

$$\mathrm{d}\quad = \text{—}\mathrm{d}\ = -\tfrac{1}{2}\ ^{(0)}_{,0}\quad\ ^2_0\exp(-2\ ^2/\quad ^2)(4\ ^2/\quad ^3)\frac{\mathrm{d}}{\mathrm{d}}\mathrm{d} \tag{A5}$$

$$= -\frac{2\ ^{(0)}_{,0}\ ^2}{(\ +\ ^2/\ )^2}\exp(-2\ ^2/\quad ^2)\mathrm{d}\ . \tag{A6}$$

Since the total number of photons is conserved this difference must be accounted for by photon flow into the disc through its cylindrical sidewall, having radius !and height d *z*. The GB is cylindrically symmetrical and so the photon flow rate perpendicular to the sidewall is independent of azimuthal angle. This means that the horizontal photon flow rate directly away from the *z*-axis (perpendicular to the disc sidewall) per unit area, $n_!^{(0)}$, is given by the solution of

$$\mathrm{d}\quad = -\int_{\text{sidewall}}\ ^{(0)}\mathrm{d}\ = -\ ^{(0)}2\quad \mathrm{d} \tag{A7}$$

and so the horizontal photon flow rate per unit area from a conventional TEM GB at radius ! from the *z*-axis is

$$^{(0)} = \ ^{(0)}_{,0}\frac{}{^2(1+\ ^2/\ ^2)^2}\exp(-2\ ^2/\quad ^2). \tag{A8}$$

As expected by symmetry, $n_{!}^{(0)} = 0$ for != 0 and also for $z = 0$. In addition, $n_{!}^{(0)}(z)$ is an odd function of z (linear for small z and anti-symmetrical in z) so that $n_{!}^{(0)}(z) = -n_{!}^{(0)}(-z)$. Since $n_{!}^{(0)} = 0$ for $z = 0$, it should be possible in principle to measure any transverse photon flux of gravitational origin (PPF) in the plane $z = 0$ without masking by BPF, but in practice it will probably be almost impossible to generate a GB with sufficient precision to suppress the BPF significantly below the level of any gravitational effects predicted. Xiang *et al.* [25] present some experimental plots of transverse power flux that agree with Eqn. (A8).

References

[1] Cruise A M 2000 *Classical and Quantum Gravity* **17**(13) 2525-2530

[2] LIGO Scientific Collaboration and Virgo Collaboration 2009 *Nature* **460** 990-994

[3] Giovannini M 1999 *Phys. Rev. D* **60**(12) 123511

[4] Giovannini M 2009 *Classical and Quantum Gravity* **26**(4) 045004

[5] Gertsenshte±n M E 1962 *Sov. Phys. JETP* **14**(1) 84-85

[6] Berlin A, Blas D, D'Agnolo R T, Ellis S A R, Harnik R, Kahn Y and Schütte-Engel J 2022 *Phys. Rev. D* **105** (11) 116011(23)

[7] Li F and Baker R M L 2007 *Int. J. Modern Phys.* B **21**(18 & 19) 3274-3278

[8] Yariv A 1989 *Quantum Electronics* (Wiley) 3rd edn chs 6 & 22, and Yariv A 1985 *Optical Electronics* (Holt, Rinehart and Winston) 3rd edn chs 2 & 13

[9] Li F-Y, Tang M-X and Shi D-P 2003 *Phys. Rev. D* **67**(10) 104008

[10] Balanis C A 1989 *Advanced Engineering Electromagnetics* (Wiley) ch 8

[11] Li F, Baker R M L, Fang Z, Stephenson G V and Chen Z 2008 *European Phys. J. C* **56** 407–423 DOI 10.1140/epjc/s10052-008-0656-9

[12] Li F, Yang N, Fang Z, Baker R M L, Stephenson G V and Wen H 2009 *Phys. Rev. D* **80** 064013

[13] Li F and Yang N 2004 *Chinese Phys. Lett.* **21** 2113-2116

[14] Li J, Lin K, Li F and Zhong Y 2011 *General Relativity and Gravitation* **43**(8) 2209–2222